Multivariate directional wavelet systems on the torus

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For regular matrices $\mathbf{M} \in \mathbb{Z}^{2\times 2}$ we consider finite dimensional spaces $V_{\mathbf{M}}^{\xi}$ containing translates of functions $\xi \in L_2(\mathbb{T}^2)$ on the pattern $\mathbf{M}^{-1}\mathbb{Z}^d \cap [-\frac{1}{2}, \frac{1}{2})^2$. The factorization $\mathbf{M} = \mathbf{J}\mathbf{N}$ with regular matrices $\mathbf{J}, \mathbf{N} \in \mathbb{Z}^{2\times 2}$ and $|\det \mathbf{J}| = 2$ leads to a decomposition of $V_{\mathbf{M}}^{\xi}$ into the direct sum $V_{\mathbf{M}}^{\xi} = V_{\mathbf{N}}^{\varphi} \oplus V_{\mathbf{N}}^{\psi}$, which was investigated in [2]. In particular, multivariate trigonometric polynomials of Dirichlet-type were used as scaling functions with corresponding wavelets.
In [1], the extension to multivariate wavelet systems with trigonometric polynomials of de la Vallée Poussin-type was developed. The construction allows for many different dilation matrices, especially shearing matrices, which can be used for anisotropic wavelet decompositions that prefer certain directions.
This talk presents a systematic way to choose the dilation matrices such that directional features of functions are revealed in certain wavelet spaces.

References

[1] Bergmann, R. and Prestin, J.: *Multivariate periodic wavelets of de la Vallée Poussin-type.* Journal of Fourier Analysis and Applications 21 (2014), 342-369.

[2] Langemann D., Prestin J.: *Multivariate periodic wavelet analysis.* Applied and Computational Harmonic Analysis 28 (2010), 46-66.