A parameter estimation of multivariate exponential sums

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The reconstruction of sparse exponential sums out of sampling values is an essential problem in signal processing. Namely, let us consider N-sparse d-variate exponential sum, $s_{\mathbf{k}} = \sum_{j=1}^{N} a_j \exp\left(-\mathrm{i}\langle \mathbf{w}_j, \mathbf{k}\rangle\right)$, where $\mathbf{w}_j \in (0,\pi]^d$, $\mathbf{k} \in \mathbb{Z}_+^d$, $\langle \mathbf{w}_j, \mathbf{k}\rangle$ denotes the inner product of $\mathbf{w}_j$ and $\mathbf{k}$, $a_1, a_2, \ldots, a_N \in \mathbb{C}\backslash\{0\}$. The aim is to determine the parameters $\mathbf{w}_j$, $j = 1, ..., N$, given finitely many samples of $s_{\mathbf{k}}$. In one-dimensional case, a solution of such a problem can be easily found by the well-known Prony method. In recent years, a lot of research has been carried out in order to obtain such a method in higher dimensions (see, e.g., [2]).

Based on the one-dimensional approach developed in [1], we propose to find the parameters $\mathbf{w}_j$, $j = 1, ..., N$, as common zeros of some special kind of d-variable polynomials. Using Cantor tuple functions allows to give the lower number of samples needed for recovery of parameters, in comparison with the other existing methods. As well combining the new approach with an autocorrelation sequence can give stable solunions in the case of noisy data.

References

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