Application of the p-adic analysis to the modelling of the capillary flow in porous medium

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This talk is dedicated to the the p-adic model of propagation of fluids (e.g., oil or water) in capillary networks in a porous random medium. Theoretical studies of the capillary motion of fluids and gases runs back to the paper by Washburn who proposed a mathematical model for this. Typical application of the capillary phenomena explored in geophysics, such as leakage in the oil pollution of soil, the capillaries form a tree. The hierarchic structure of a system of capillaries is mathematically modeled by endowing trees of capillaries with the structure of an ultrametric space.

We introduce and study an inhomogeneous Markov process describing the penetration of fluid into a porous random medium. Mathematically such trees can be represented as ultrametric (non-Archimedean) spaces. This ultrametric representation will be explored, where we study the idealized model based on regular p-trees, for which p branches come forward from each vertex. By the purely mathematical reason it is convenient to restrict considerations to the case of prime $p > 1$. The crucial point is that such trees can be endowed with the natural algebraic structure of the number field: addition, subtraction, multiplication and division (in the case of the prime p) are well defined on the set of branches of a p-tree.

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