Optimal recovery of Cauchy type integrals in the unit disk

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Let K be the set of holomorphic functions f in $\mathbb{D} := \{z : |z| < 1\}$ which is represented by Cauchy type integral

$$f(z) = \frac{1}{2\pi i} \int_{\mathbb{T}} \frac{h(t)}{t - z} dt, \quad z \in \mathbb{D},$$

where $h : \mathbb{T} \to \mathbb{C}$ is essentially bounded function on $\mathbb{T} := \{t : |t| = 1\}$ and ess $\sup_{t\in\mathbb{T}} |f(t)| \leq 1$.

For distinct points $\mathbf{a} := \{a_0, a_1, \ldots, a_{n-1}\}$, $n \in \mathbb{N}$, in $\mathbb{D}$ consider the quantity

$$\mathscr{E}_n(K; \mathbf{a}; z) := \inf_{\mu_j} \sup_{f\in K} \left| f(z) - \sum_{j=0}^{n-1} f(a_j)\mu_j(z) \right|, \quad z \in \mathbb{D},$$

where infimum is taken over all linearly independent systems of continuous functions in $\mathbb{D}$.

Theorem. *Let $n \in \mathbb{N}$, $\mathbf{a}$ be as above and*

$$B_j(t) := \prod_{k=0}^{j-1} \frac{-|a_k|}{a_k} \cdot \frac{t - a_k}{1 - t\overline{a_k}}, \quad j = 1, 2, \ldots, n,\ t \in \mathbb{D}.$$

Then for each $z \in \mathbb{D}$,

$$\mathscr{E}_n(K; \mathbf{a}; z) = \frac{|B_n(z)|(1 - |z|^2)}{1 - |zB_n(z)|^2} \int_{\mathbb{T}} \frac{\left|1 - \overline{zB_n(z)}tB_n(t)\right|}{|1 - \overline{z}t|^2} \frac{|dt|}{2\pi}.$$