

Nikolskii-type estimate for nearly copositive approximation of continuous on an interval functions

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If a continuous on a closed interval function f changes its sign at s points $y_i : -1 < y_s < y_{s-1} < \dots < y_1 < 1$, then for each $n \in \mathbb{N}$, greater then some constant $N(k, y_i)$ depending only on $k \in \mathbb{N}$ and $\min_{i=1, \dots, s-1} \{y_i - y_{i+1}\}$, we construct an algebraic polynomial P_n of degree $\leq n$ such that: P_n has the same sign as f , everywhere except, perhaps, small neighborhoods of the y_i :

$$(y_i - \rho_n(y_i), y_i + \rho_n(y_i)), \quad \rho_n(x) := 1/n^2 + \sqrt{1 - x^2}/n,$$

$P_n(y_i) = 0$ and

$$|f(x) - P_n(x)| \leq c(k, s) \omega_k(f, \rho_n(x)), \quad x \in [-1, 1],$$

where $c(k, s)$ is a constant depending only on k and s , and $\omega_k(f, \cdot)$ is the modulus of continuity of the k -th order of the function f .